Liquidity as a Choice Variable: A Lesson from the Japanese Government Bond Market

Jacob Boudoukh
Robert F. Whitelaw
New York University

In Japan, almost identical government bonds can trade at large price differentials. Motivated by this phenomenon, we examine the issue of the value of liquidity in markets for riskless securities. We develop a model of an issuer of bonds, a market maker, and heterogeneous investors trading in an incomplete market. We show not only that divergent prices for similar securities can be sustained in a rational expectations equilibrium, but also that this divergence may be optimal from the perspective of the issuer. Price segmentation is possible because agents have a desire to trade, but short-sale restrictions limit their trading strategies and prevent them from forcing bond prices to be equal. Restricting the form of market making to exclude price competition and unregulated profit maximization is also necessary to sustain price segmentation. The optimality of segmentation from the issuer's standpoint arises because of the issuer's ability to charge for the liquidity services provided to the investors.

We thank Anat Admati, Bruce Grundy, Lars Hansen, Allan Kleidon, Paul Pfleiderer, Tom Sargent, Myron Scholes, Ken Singleton, and Avanidhar Subrahmanyam; participants in presentations at the 1991 WFA meetings and Stanford University; the editor, Chester Spatt; and an anonymous referee for helpful suggestions and discussions. We also acknowledge the financial support of the graduate program at Stanford University, Graduate School of Business, where most of this project was completed. Address correspondence to Jacob Boudoukh, New York University, Stern School of Business, 44 West 4th Street, New York, NY 10012-1126.

The Review of Financial Studies 1993 Volume 6, number 2, pp. 265–292

> *The "benchmark" effect: over 90 percent of cash market trading in [Japanese government bonds] can take place in a single issue, termed the "benchmark" . . . with yields of over 70 basis points below the yields of issues with almost identical terms.* Mason (1987, p. 48)

How can we reconcile the fact that two similar assets trade at such different prices with the notion of efficient markets?

Traditional asset pricing models, where markets exhibit no frictions or restrictions, are clearly unable to sustain this phenomenon, yet it has been a feature of the Japanese government bond (JGB) market for many years. Over the last decade, the yield spread between a basket of side issues and the various benchmark issues has averaged 40–60 basis points, with a spread as high as 100 basis points or more at times during 1986 and 1987 [Mason (1987, p. 61), Boudoukh and Whitelaw (1991)]. The same phenomenon, albeit on a smaller scale, exists in the United States in the short- and long-maturity bond markets. A long-maturity (typically 30 years) bond, the "on-the-run" issue, can trade at prices that imply yields 10 basis points lower than those on other bonds with similar maturities, coupons, and call provisions. In the short-maturity (less than three months) market, Treasury bills are often more liquid and more expensive than Treasury notes with similar maturities [Kamara (1990), Amihud and Mendelson (1991)].

The feature that the benchmark issue, "on-the-run" bonds, and Treasury bills have in common is that they are heavily traded relative to similar traded securities, suggesting that it is not an intrinsic property of the security that makes it more valuable but rather the very existence of heavy trade in the specific bond issue. In fact, the benchmark issue accounts for 95 percent of total trading in all JGBs originally issued with 10-year maturities [Boudoukh and Whitelaw (1991)],[1] whereas the comparable ratio for the most active 30-year bond in the United States is 10 percent [Sargen et al. (1986)]. Since the benchmark bond in Japan is officially designated,[2] one might conjecture that the Japanese government is intentionally segmenting the market in terms of liquidity.

[1] Original issues with 10-year maturities account for the majority of new issues of JGBs. Longer and shorter maturities exist but are relatively new to the market.

[2] The mechanism for choosing the benchmark issue is not precisely defined. Generally, a bond with approximately eight to nine years to maturity, with a large outstanding total par value, and that trades close to par is selected and remains the benchmark bond for a period of about one year. During transitions between benchmark bonds, much speculation on the identity of the new benchmark occurs. Once designated, the benchmark is likely to remain the benchmark for nine months or more.

We develop a heterogeneous-agent incomplete-market model that enables us to suggest an explanation for the benchmark effect. Divergence of prices can be sustained for bonds with identical payoffs that differ only in their relative liquidity. Furthermore, we show that under certain conditions the issuer would *choose* to create a segmented market. Throughout, we examine the assumptions that underlie our model, motivate their necessity, and relate them to the institutional features of the JGB market.

We consider a three-period economy with a continuum of utility-maximizing agents who are heterogeneous in their endowments. The agents can transfer wealth intertemporally only by trading in two issues of zero coupon bonds, which pay one unit of the consumption good in the third period. The model's equilibria are categorized as *symmetric* equilibria, in which the prices of the two securities are identical, and *segmented* equilibria, in which the prices differ.

The issuer is assumed to minimize the discounted face value of the debt issued less discounted market-maker revenue, subject to a fixed revenue requirement, taking as given the agents' optimal trading behavior and demands. We assume that the issuer can regulate the market maker or that they act cooperatively. This assumption is plausible for Japan, where a fixed underwriting syndicate is used for all new issues, and the Ministry of Finance is believed to be able to impose terms on the group. Given a specific set of parameters, we show the optimality (from the issuer's and market maker's standpoint) of an equilibrium in which one bond is liquid and sells for a higher price than its illiquid counterpart.

The value of liquidity results from agents' uncertainty about their endowments and, therefore, about their trading needs. The issuer optimally engages in price discrimination and extracts consumer surplus, exploiting the agents' precautionary demand for liquidity. A bond consists of two components: the claim to the final payoff and the extent of its liquidity before maturity. Rather than discriminating between different types of investors, the issuer in this case segments markets across these two components. We extend the analysis to consider cases in which the issuer can determine the relative supply of the bonds; segmentation of markets along the dimension of liquidity emerges as globally optimal.

Which features of the model, other than agents' heterogeneity, make a segmented equilibrium possible? Perhaps the most important feature is that agents are not allowed to sell bonds short. We show that costless short sales are inconsistent with any price discrepancies between the bonds; therefore, a necessary condition for any price segmentation to occur is that short sales be prohibited, restricted, or costly. This restriction is consistent with the JGB market. Another

important feature is the form of market making. In our model, if market making is competitive, then period-2 bid–ask spreads on both bonds are driven to zero and price differences between the two bonds are impossible to sustain. When we consider a regulated market maker, which is consistent with the institutional structure of the JGB market, price segmentation arises as the optimum.

Our work combines three central issues related to liquidity: the optimization problem of the agents, the optimization problem of the issuer and the market maker, and the concentration of trade. In the spirit of Grossman and Laroque (1990), Garbade and Silber (1976), Lippman and McCall (1986), and Amihud and Mendelson (1986), agents choose optimal trading strategies, taking transaction costs as given. The problem of the issuer and the market maker, in turn, is viewed as one of choosing posted bid and ask prices, as in Ho and Stoll (1981), Amihud and Mendelson (1980), and Glosten and Milgrom (1985). Concentration of trade—studied in Admati and Pfleiderer (1988), Foster and Viswanathan (1990), and Pagano (1989)—arises endogenously in our model. This article resembles that of Pagano (1989) in that the trade is concentrated in a given market rather than at a given point in time. It differs in that agents are not restricted from trading in both markets.

The article is organized as follows. In Section 1, the basic model is presented, and market restrictions are examined and related to the institutional features of the JGB market. In Section 2, we solve the agents' maximization problem given a set of prices. In Section 3, we consider the price-setting problem as faced by the issuer and the market maker. We present results on the optimality of segmented markets and a discussion of the effect of varying relative supplies and endowment volatility. In particular, as endowment volatility increases (in a mean-preserving-spread sense), the liquidity premium (the difference between the period-1 bid prices of the liquid and illiquid bonds) increases. In Section 4, we consider the case in which the market maker maximizes revenue independently of the issuer. When the magnitude of the bond issue is small relative to the endowments, the equilibrium that emerges is symmetric. We conclude the article in Section 5.

1. The Model

1.1. The setup

Consider a three-period (1,2,3) endowment economy in which only two pure discount bonds are traded. Both bonds are issued in period

1, mature in period 3, are riskless, and pay one unit of consumption at maturity (all prices are denominated in units of the consumption good). We designate the bonds as liquid (L) and illiquid (I). Although the bonds have different designations, we do not exclude cases in which they trade at identical prices. In these cases, the designations are irrelevant. They are in positive per capita net supply of $k\Omega$ ($k > 0$) and Ω. The prices are denoted p^k_{ij}, where $i = a, b$ denotes ask or bid price, $j = 1, 2$ denotes period 1 or 2, and $k = I, L$ denotes illiquid or liquid (e.g., p^L_{b2} is the period-2 bid price of the liquid bond). Without loss of generality, the bond with the lower ask price in period 1 is designated as the illiquid bond; in other words, $p^I_{a1} \leq p^L_{a1}$.

There is a continuum of agents indexed on [0, 1] who maximize expected, time-separable, discounted, von Neumann–Morgenstern utility. The agents are not permitted to sell bonds short. In addition, they cannot make markets or hedge by forming coalitions; the only way of transferring wealth across time is through bonds. The agents are divided into two groups, distinguished by their endowments of the consumption good in each of the last two periods. In the initial period the agents do not know into which group they fall. They receive identical endowments, denoted Y_1, and hence take identical initial positions in the bond market, h^I_1 illiquid bonds and h^L_1 liquid bonds. At the beginning of the second period agents are informed of their endowment and therefore realize their type: buyers (B), who purchase bonds in the second period, or sellers (S), who sell bonds in the second period. Buyers receive the endowments Y^B_2 in period 2 and Y^B_3 in period 3, and hold h^I_{B2} illiquid bonds and h^L_{B2} liquid bonds. For sellers the notation is Y^S_2, Y^S_3, h^I_{S2}, and h^L_{S2}, respectively.

The agents are designated such that the marginal rate of substitution between periods 2 and 3 of the buyers exceeds that of the sellers if no bonds are traded in period 2. This assumption ensures that the sellers always have more incentive than the buyers to sell bonds in period 2. The probability q of being a buyer is known. The endowments of the sellers and the buyers are also known in period 1.

In the first period the issuer sells the total supply of bonds at ask prices that will clear the markets. In period 2 all the trade goes through a market maker at his posted bid and ask prices. The market maker holds no inventory, and he must accept all orders at his posted prices; therefore, he must set prices to equate supply and demand in the markets for both securities. The market maker has no modeled consumption needs and is interested only in period-2 revenues. In the third period the bonds mature and their payoff is consumed in full. Table 1 shows the notation for the parameters, all of them being nonnegative.

It is useful to define the notion of equilibrium in this model.

Table 1
Bond prices and supplies and agents' endowments and holdings

	Payoffs			Prices (bid/ask)			Per capita supply
	Period 1	Period 2	Period 3	Period 1	Period 2	Period 3	
Liquid (L)	0	0	1	p^L_{a1}	p^L_{b2}, p^L_{a2}	1 1	$k\Omega$
Illiquid (I)	0	0	1	p^I_{a1}	p^I_{b2}, p^I_{a2}	1 1	Ω
	Endowments			Position (liq./illiq.)			Fraction in the population
	Period 1	Period 2	Period 3	Period 1	Period 2	Period 3	
Seller (S)	Y_1	Y^S_2	Y^S_3	b^L_1, b^I_1	b^L_{S2}, b^I_{S2}	0 0	$1 - q$
Buyer (B)	Y_1	Y^B_2	Y^B_3	b^L_1, b^I_1	b^L_{B2}, b^I_{B2}	0 0	q

Definition 1. An *equilibrium* is a set of positive prices such that (i) the supply of bonds equals the demand for bonds (markets clear) in periods 1 and 2, and (ii) all agents maximize their expected, discounted utility.

In such a model, there will often be an infinite number of equilibria that do not differ in any substantive fashion. For example, if there exists an equilibrium in which no trade occurs in period 2 in the illiquid bond with ask price $p^I_{a2} = \hat{p}$, then for all $p^I_{a2} > \hat{p}$ we can achieve the same consumption, revenues, and so forth. We will call all these equilibria equivalent.

Definition 2. Equivalent equilibria are equilibria that support identical consumption streams and trading strategies for the agents and identical revenue streams for the issuer and the market maker, but differ in the equilibrium prices.

To eliminate this multiplicity of equilibria, we will confine our attention in each case to the equilibrium, from the set of equivalent equilibria, with the lowest ask prices and highest bid prices.

In equilibrium, the period-2 ask prices of the bonds are identical ($p^I_{a2} = p^L_{a2}$), remembering that we ignore equivalent equilibria with higher ask prices. These prices are equal because the bonds are a claim to identical future cash flows. In addition, in equilibrium the period-2 bid price of the illiquid bond is less than or equal to the period-2 bid price of the liquid bond ($p^I_{b2} \leq p^L_{b2}$). This inequality holds because of the relationship between the ask prices in period 1 and the fact that both bonds have identical payoffs in the final period.[3]

[3] Although motivated by the JGB market, the model can be applied to other markets in which differential liquidity creates a wedge between prices of similar assets. Kamara (1990), for example,

1.2 Classification of equilibria

The equilibria in the model can be divided into two categories: equilibria with identical prices (*symmetric*) and equilibria in which either bid or ask prices differ at some point in time (*segmented*). In a segmented equilibrium both the period-1 ask prices and period-2 bid prices will differ across bonds; that is, there is no equilibrium in which $p^L_{a1} = p^L_{a1}$ and $p^L_{b2} \neq p^L_{b2}$, or $p^L_{a1} \neq p^L_{a1}$ and $p^L_{b2} = p^L_{b2}$. The posted prices determine which equilibrium will occur. The symmetric equilibria can be further classified based on period-2 trade as follows.

Equilibrium 1. No trade occurs—if the bid price is set low enough and the ask price is set high enough, none of the agents will wish to trade.

Equilibrium 2. Trade occurs, but the constraint (for the sellers) on nonnegative holdings is not binding—for certain prices the sellers will wish to trade period 3 consumption for period 2 consumption by selling bonds, and the buyers will wish to pursue the reverse strategy.

Equilibrium 3. Trade occurs and the constraint on nonnegative holdings is binding—as the bid price rises, the sellers wish to sell more of their holdings until the constraint on short sales begins to bind.

Segmented markets can also be classified on the basis of period-2 trading behavior.

Equilibrium 4. All the liquid bonds held by the sellers are traded, but no illiquid bonds are traded—the bid price on liquid bonds is set so that the constraint on nonnegative holdings binds, and the bid price on the illiquid bonds is low enough so that the sellers do not wish to sell any of their holdings of these bonds.

Equilibrium 5. All the liquid bonds held by the sellers are traded, and some illiquid bonds are traded—the nonnegative holdings constraint on the liquid bonds for the sellers binds, and the bid and ask prices on the illiquid bond are set so that trade occurs.

Equilibrium 6. All the liquid and illiquid bonds held by the sellers are traded—the period-2 bid prices on both bonds are high enough

---

compares the yields on U.S. Treasury bills (TBs) to those on Treasury notes (TNs) in their final coupon period. Assuming that search costs and the bid–ask spread in the TN market are higher than those in the TB market, he shows that in equilibrium TN prices must be lower than TB prices. Kamara's empirical results strongly support this statement regarding the bid prices, but he cannot reject that the ask prices are the same. Our model predicts exactly this: in equilibrium the ask prices in the second period will be the same, and the differential liquidity will strike a wedge between the bid prices in the second period. These results are consistent with the observation that the buyers of these bonds are likely to hold them to maturity.

Table 2
Equilibria and trading volume

Equilibrium	Type	Period-2 trade	
		Liquid	Illiquid
1	Symmetric	None	
2	Symmetric	Some	
3	Symmetric	All	
4	Segmented	All	None
5	Segmented	All	Some
6	Segmented	All	All

to induce the sellers to liquidate their holdings, but these prices are not identical.

These definitions are summarized in Table 2. The Period-2 trade column refers to trade in the bonds held by the sellers coming into period 2 (i.e., the bonds they purchase in period 1). The buyers hold all bonds they purchase in period 1 until maturity in period 3.

In segmented equilibria the period-1 ask prices and the period-2 bid prices of the two bonds differ. The liquid bond has a higher ask price in period 1 because it can be sold at a higher bid price in the second period. It is important to note that segmented equilibria in which only some of the liquid bonds held by the sellers are traded cannot be supported. Agents will not pay, in period 1, for liquidity that they will never use. There is no distinction between a liquid bond and an illiquid bond if both are certain to be held to maturity; therefore, in a rational expectations equilibrium, both securities must have identical prices.

1.3. Market restrictions

In this section we provide a critical evaluation of the main features of the model. In particular, we examine the effect of such restrictions as short-sale constraints, no market-maker inventory, the noncompetitive structure of market making, and the set of tradable securities. We conclude that for price segmentation to be sustainable, short-sale constraints and a noncompetitive market-making mechanism are essential.

Perhaps the most crucial assumption is that agents are not allowed to sell bonds short. Without this assumption, however, the following result holds.

Theorem 1. *If the agents can costlessly sell bonds short in period 2 at the bid price (without any restrictions), then in equilibrium* $p^I_{b2} = p^L_{b2}$ *and* $p^I_{a1} = p^L_{a1}$.

Proof. See Appendix A.

The theorem states that costless short sales are inconsistent with any price discrepancies between the bonds. The proof relies on the fact that a rational agent who wishes to increase his consumption in period 2 will sell the bond with the higher bid price regardless of his portfolio holdings. Consequently, if both bonds are traded, they must have equal period-2 bid prices. If one or both bonds are not traded, the elimination of equivalent equilibria establishes the result.

This theorem shows that, in our model, a necessary condition for price segmentation to occur is that short sales be prohibited, restricted, or costly. If, for example, we assume that short sellers incur a cost equal to 5 percent of their proceeds, a result similar to Theorem 1 with the limitation that $p^l_{b2} \geq p^l_{b2} \geq .95p^l_{b2}$. Other schedules of short-selling costs can be similarly accommodated. This suggests that large price spreads between similar securities of different liquidities can only be supported when short sales are severely restricted or extremely costly. In the JGB market such conditions exist. In contrast, the United States has a more organized and less costly mechanism for short selling, although there are costs that prevent traders from taking large short positions.

Another important feature of our model, in addition to short-sale constraints, is the market-making structure. Two issues are of particular importance: the lack of market-maker inventory and the absence of price competition. First, consider the fact that the market maker has a role only during period 2, and therefore, by definition, carries no inventory. This assumption coincides with the institutional features of the JGB market. Inventory costs are high, partly because of the inability to hedge adequately interest rate risk using either short positions or the futures market. As a result, market makers do not maintain large positions over long time periods, especially in the less liquid securities [Mason (1987)]. While partial-equilibrium, multiperiod, inventory models address the market maker's problem in more generality,[4] they fail to capture the interaction between the issuance problem and the investment problem, an issue on which we focus.

Second, we restrict our analysis to noncompetitive market making. If market making is competitive, period-2 bid–ask spreads on both bonds are driven down to the market maker's marginal cost of transacting. Price competition drives market makers to continually undercut each other on trades until profits are eliminated.[5] Barring an ad

[4] See, for example, Amihud and Mendelson (1980) and Ho and Stoll (1981, 1983) for inventory-risk models, and see Tuckman and Vila (1992) for a holding-cost model.

[5] This result holds, in our model, when two or more market makers compete noncooperatively through posted prices.

hoc assumption of differential marginal trading costs in the two bonds, segmentation is not sustainable. We consider both a regulated market maker and a monopolist market maker, although the former case is more consistent with the institutional features of the JGB market. In a richer framework with inventory costs, market segmentation may arise endogenously, even in a competitive market-making environment, as a result of differential trading volumes.

A final feature of the model is the restriction of the set of tradable securities to two two-period bonds. However, the possible introduction of a one-period bond does not overturn the results regarding the existence and optimality of segmented markets. A sequence of two one-period bonds can replace the liquid two-period bond at the optimum derived in Section 3 without affecting the results, except that segmentation occurs across bonds with different maturities.[6] The one-period bonds are, by definition, liquid, and the issuer still finds it optimal to issue illiquid two-period bonds. Note that in this model the agents face type uncertainty but no aggregate uncertainty; therefore, interest rates are nonstochastic. In a model with stochastic interest rates, the equivalence between a long liquid bond and a rollover short-term bond will no longer hold. In such a model the long liquid bond will possess both a term and a liquidity premium. However, the liquidity premium may be smaller than if no short-term security exists, consistent with the observed difference between the U.S. Treasury market and the JGB market.

2. Solving the Agents' Optimization Problem

2.1. The problem

Consider first the optimal trading strategies of the agents for a given set of prices. Assuming that we are in a selected class of equilibria from Section 1.2, the first-order conditions from utility maximization provide restrictions on the set of possible prices. The issuer and market maker then set prices, given the agents' optimal mapping from prices to bond holdings, in order to minimize costs within certain constraints. We look at the agents' problem and the resulting price restrictions in this section. In Section 3, we study the problem of price determination.

The agents maximize the expected value of the sum of discounted utility. Their problem in the initial period is

[6] The restricted set of securities is more representative of the JGB market, where 10-year bonds account for the majority of new issues and short-term bonds are rare.

$$\max_{c_t, b_t^j} E_1\left[\sum_{t=1}^{3} \beta^{t-1} u(c_t)\right]$$

$$\text{s.t. } c_1 \le Y_1 - h_1^I p_{a1}^I - h_1^L p_{a1}^L,$$

$$c_2 \le Y_2 - 1_I(h_2^I - h_1^I)p_{a2}^I - 1_L(h_2^L - h_1^L)p_{a2}^L$$

$$- (1 - 1_I)(h_2^I - h_1^I)p_{b2}^I - (1 - 1_L)(h_2^L - h_1^L)p_{b2}^L,$$

$$c_3 \le Y_3 + h_2^I + h_2^L,$$

$$h_1^I \ge 0, \quad h_1^L \ge 0, \quad h_2^I \ge 0, \quad h_2^L \ge 0,$$

where $t = 1,2,3$, $i = 1,2$, $j = I,L$, and (P1)

$$1_I = \begin{cases} 1, & \text{if } h_2^I \ge h_1^I, \\ 0, & \text{otherwise;} \end{cases} \qquad 1_L = \begin{cases} 1, & \text{if } h_2^L \ge h_1^L, \\ 0, & \text{otherwise.} \end{cases}$$

The expectation, $E_1[\cdot]$, is taken over the distribution of agent types because initially agents do not know if they are buyers or sellers. In period 1, agents can either consume their endowments or invest in bonds. In period 2, agents adjust their portfolios of bonds subject to the short-sale constraint; they consume their endowments plus net investment. In the final period the agents consume their endowments plus the value of maturing bonds.

Because of the complexity of working with first-order conditions for problems containing indicator functions such as 1_I and 1_L, we will look for solutions within each class of equilibria and later find parameter values for which these solutions are supported. Perhaps the most interesting equilibria are those in which period-2 trade takes place in both bonds (Equilibrium 5). The solution to this problem is outlined in the next subsection and is detailed in Appendix B.

2.2. Equilibrium 5

Under the assumption that this class of equilibria occurs and that the agents are carrying holdings h_1^I and h_1^L into period 2, the period-2 problem of the sellers becomes

$$\max_{b_{S2}^I} u(c_2) + \beta u(c_3) \quad \text{s.t. } c_2 \le Y_2^S + h_1^L p_{b2}^L - (h_{S2}^I - h_1^I)p_{b2}^I,$$

$$c_3 \le Y_3^S + h_{S2}^I. \qquad \text{(P2S)}$$

Recall that the agents realize their type prior to trade in period 2 and that the sellers sell all of their liquid bonds and some of their illiquid bonds in period 2 in this equilibrium. Consequently, their decision variable is their period-2 holdings of the illiquid bond, h_{S2}^I, and the problem has a solution h_{S2}^{I*}.

Under the same assumptions, the period-2 problem for the buyers becomes

$$\max_{b_{B2}^I, b_{B2}^L} u(c_2) + \beta u(c_3) \quad \text{s.t. } c_2 \leq Y_2^B - [(h_{B2}^L + h_{B2}^I) - (h_1^L + h_1^I)]p_{a2},$$

$$c_3 \leq Y_3^B + (h_{B2}^L + h_{B2}^I). \tag{P2B}$$

In formulating this problem we have used the fact that in equilibrium the ask prices of the bonds are equal ($p_{a2}^I = p_{a2}^L \equiv p_{a2}$). This equality also means that the portfolio holdings of the buyers (h_{B2}^I and h_{B2}^L) are not separately identified because the purchasers of bonds in period 2 make no distinction between the two types of bonds. Therefore, we maximize over their sum, $h_{B2} \equiv h_{B2}^I + h_{B2}^L$, to get the optimal total holdings of the buyers, h_{B2}^*.

Substituting these solutions back into the initial-period problem (P1), we have

$$\max_{h_1^I, h_1^L} u(Y_1 - h_1^I p_{a1}^I - h_1^L p_{a1}^L) + \beta q u(Y_2^B - [h_{B2}^* - (h_1^I + h_1^L)]p_{a2})$$

$$+ \beta(1-q)u(Y_2^S + h_1^L p_{b2}^L - (h_{S2}^{I*} - h_1^I)p_{b2}^I)$$

$$+ \beta^2 q u(Y_3^B + h_{B2}^*) + \beta^2(1-q)u(Y_3^S + h_{S2}^{I*}),$$

where the expectation is written explicitly in terms of the probability of being a buyer (q) or a seller ($1-q$). The two first-order conditions from this problem implicitly define the agents' optimal period-1 holdings of the liquid and illiquid bonds, h_1^{I*} and h_1^{L*}.

In equilibrium, for supply to equal demand in each period, the following three conditions must also hold:

$$h_1^{I*} = \Omega, \tag{1}$$

$$h_1^{L*} = k\Omega, \tag{2}$$

$$q h_{B2}^* + (1-q)h_{S2}^{I*} = (1+k)\Omega. \tag{3}$$

The first two conditions ensure that markets clear in period 1, and the third condition ensures that markets clear in period 2. There is only a single market-clearing condition in period 2 because the volume of trade in the liquid bond is fixed given that we are in Equilibrium 5.

Liquidity is valued in this model because agents, when realizing their type (in period 2), wish to trade. Therefore, they are willing to pay a premium for a liquid bond in period 1. The agents' precautionary demand for liquidity enables the issuer to engage in price discrimination and extract consumer surplus.

The four first-order conditions for h_1^{I*}, h_1^{L*}, h_{S2}^{I*}, and h_{B2}^{*} and the three equilibrium conditions reduce to three equations in the five unknown prices. We can specify two of the five prices, and the other three prices will be determined in equilibrium. In theory, we could fix any pair, but the approach we take is to set p_{b2}^{L} and p_{b2}^{I}, the bid prices in period 2.[7] Under log utility,[8] the solution for the ask price in period 2 is

$$p_{a2} = q\beta Y_2^B p_{b2}^I / \{p_{b2}^I[(1 + k)\Omega + \beta(1 - q)k\Omega + (1 - q)Y_3^S + qY_3^B] - (1 - q)\beta(Y_2^S + k\Omega p_{b2}^L)\}. \tag{4}$$

The period-1 ask prices of the liquid and illiquid bonds are

$$p_{a1}^L = \frac{Y_1(E + F^L)}{1 + \Omega(E + F^I) + k\Omega(E + F^L)}, \tag{5}$$

$$p_{a1}^I = \frac{Y_1(E + F^I)}{1 + \Omega(E + F^I) + k\Omega(E + F^L)}, \tag{6}$$

where

$$E = \frac{\beta(1 + \beta)qp_{a2}}{Y_2^B + [Y_3^B + (1 + k)\Omega]p_{a2}},$$

$$F^I = \frac{\beta(1 + \beta)(1 - q)p_{b2}^I}{Y_2^S + k\Omega p_{b2}^L + (Y_3^S + \Omega)p_{b2}^I},$$

$$F^L = \frac{\beta(1 + \beta)(1 - q)p_{b2}^L}{Y_2^S + k\Omega p_{b2}^L + (Y_3^S + \Omega)p_{b2}^I}.$$

We need to keep in mind that the choice of period-2 bid prices must be made in order to keep us in the selected class of equilibrium. The restrictions that the sellers sell some, but not all, of their illiquid bonds ($0 < h_{S2}^{I*} < \Omega$) and all of their liquid bonds ($h_{S2}^{L*} = 0$) imply the following price ranges:

$$\frac{\beta(Y_2^S + k\Omega p_{b2}^L)}{Y_3^S + \Omega} < p_{b2}^I < \frac{\beta(Y_2^S + k\Omega p_{b2}^L)}{Y_3^S - \beta\Omega}, \tag{7}$$

$$p_{b2}^L \geq \frac{\beta Y_2^S}{Y_3^S + (1 - \beta k)\Omega}. \tag{8}$$

In addition to (7) and (8), we impose the restrictions $p_{b2}^I \leq p_{a2}, p_{b2}^L$

[7] This choice is merely for methodological convenience. Any other pair will lead to equations in which higher powers of the unknown prices appear, making solutions more difficult to find.

[8] Qualitatively similar results can be derived under power utility. We resort to the log utility case only because closed forms are simpler to attain.

$\leq p_{a2}$, and that all prices are positive. These restrictions also apply to the other equilibria (solutions are available on request).

Although the solutions for the prices in (4)–(6) are cumbersome, it is still possible to extract some intuition for the relation between endowments and prices from these expressions. The period-2 ask price is positively related to the period-2 endowment of the buyers and negatively related to their period-3 endowment. As the desire of the buyers to substitute consumption from period 2 to period 3 increases, the price (return) increases (decreases). This relation captures the demand side of the market. The supply side of the period-2 market for bonds depends on the sellers. The supply of bonds decreases in the period-2 endowment of the sellers and increases in their period-3 endowment; therefore, in order to clear the market, the period-2 ask price is positively related to Y_2^s and negatively related to Y_3^s. The period-1 ask prices of the two bonds are increasing in Y_1 because the greater the period-1 endowment, the greater the desire to substitute period-1 consumption for later consumption.[9]

Given prices, it is possible to compute the volume of trade in each bond, market-maker revenues, issuer revenues, consumption, and utility. Further discussion of the properties of these variables and of prices in equilibrium is deferred until the numerical example in the following section. The complexity of the solution does not permit analytical results to be derived without imposing constraints on the parameters of the model. This complexity arises because changes in period-2 bid prices, for example, alter optimal trading strategies in period 2 and ask prices in periods 1 and 2 through changes in the marginal rates of substitution.

3. Price Segmentation and the Value of Liquidity

3.1. Solving the joint problem of the issuer and the market maker

We saw in Section 2 that period-2 bid prices could be used to determine which equilibrium obtains. The obvious question is then, which equilibrium will be chosen by those players who can determine prices? In this section, we assume that the market maker and issuer act cooperatively. The issuer needs to raise a given amount of capital, and prices and bond supplies are set so as to minimize the face value of the bonds issued minus the amount of market-maker revenue in period 2, adjusted by factors that account for their different timing. This goal

[9] For this analysis we assume that the supply of bonds does not depend on the endowment processes or bond prices. In the next section we consider a problem in which period-1 issuer revenue is fixed, and, therefore, the supply changes in response to price changes.

is consistent with the issuer being able to extract market-maker revenues through the use of taxation or by the imposition of fees.

When the government is the issuer, the powers of taxation and certification make control of the market maker plausible. This is especially true in Japan, where for many years a fixed underwriting syndicate and selling group was used for all new issues. A bond issue is authorized by the Ministry of Finance and implemented by the Bank of Japan. They then negotiate the coupon and the size of the new issue with the syndicate, consisting of city, credit, regional, agricultural, trust and mutual banks, life insurance companies, and securities companies. Membership in this group was essential for full participation in the financial markets. Consequently, the Ministry of Finance could impose terms, using a threat of expulsion from the syndicate,[10] on the small group of market makers.

We solve a single minimization problem for the issuer and market maker. The objective function to be minimized is

$$W \equiv \Lambda_1(1 + k)\Omega - \Lambda_2\Pi_{MM} - X, \tag{9}$$

subject to the constraint $p^L_{a1}\Omega + p^L_{a1}k\Omega = X$, where X is the total per capita capital needed, Λ_1 and Λ_2 are the weights or the time discount factors, and Π_{MM} is the per capita market maker revenue in period 2. The market maker revenue is the bid–ask spread times the per capita volume of trade in each bond.

This formulation is quite general. As the weight on market-maker revenue increases relative to the weight on the total face amount of the bonds, the problem becomes one of maximizing market-maker revenue. This case is considered in detail in Section 4. As the relative weight on market-maker revenue decreases, the problem becomes one of minimizing the face amount of the bonds.

Since the cash flows occur in different periods and the issuer and market maker are assumed to act cooperatively, the discount rates in this section are taken to be the prices of one- and two-period discount bonds. In the pure exchange economy with logarithmic utility and average per capita endowments, these bond prices are $\Lambda_1 = \beta^2 Y_1/Y_3$, $\Lambda_2 = \beta Y_1/Y_2$, where $Y_3 \equiv qY_3^B + (1 - q)Y_3^S$ and $Y_2 \equiv qY_2^B + (1 - q)Y_2^S$.

It is prohibitively complicated to calculate the face amount of the bonds issued and market-maker revenue for each class of equilibria in closed form and to optimize analytically. Instead, we use a numerical analysis to illustrate the major results. We use the following values as the base case set of parameters:

[10] In the United States, bonds are also sold through a limited group of primary dealers. However, prices are determined in a competitive auction.

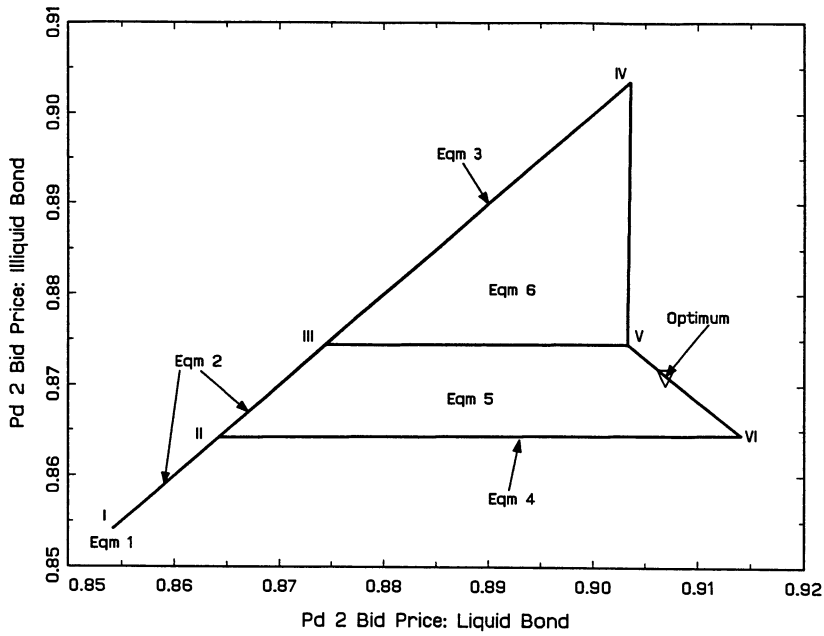

Figure 1
Feasible equilibria
Feasible equilibria by type as classified by period-2 bid prices for the parameter values $X = 1$, $\beta = .90$, $q = .5$, $Y_1 = 100$, $Y_2^S = 98$, $Y_2^B = 102$, $Y_3^S = 102$, $Y_3^B = 98$, $k = 1$. The optimum point minimizes the objective function W. Details regarding volume, market-maker revenue, the value of the objective function, and the agents' utility are in Table 3.

$$X = 1, \quad \beta = .90, \quad q = .5,$$
$$Y_1 = 100, \quad Y_2^S = 98, \quad Y_2^B = 102, \quad Y_3^S = 102, \quad Y_3^B = 98.$$

The total amount to be raised and the endowments are chosen so that the issuance of bonds does not have a major effect on the marginal rates of substitution of the agents. When the capital required is of the same order of magnitude as the endowments, then the bonds cause large amounts of consumption to be moved from period 1 to period 3. The fraction of each type of agent and endowment stream are chosen so as to avoid assuming any asymmetry up front.

3.2. The optimality of segmentation

We first look for equilibria of all types that satisfy the capital constraint and for which there is an equal supply of liquid and illiquid bonds (i.e., $k = 1$). These feasible equilibria are illustrated graphically in Figure 1. The equilibria are identified (uniquely) by the second-period bid prices of the liquid and illiquid bonds. Notice that all the equilibria lie below the 45° line, since it is assumed that the period-1 ask price of the liquid bond exceeds that of the illiquid bond (p_{a1}^l

Table 3
Selected equilibria

	Ω	$k\Omega$	p^I_{a1}, p^L_{a1}	p^I_{b2}, p^L_{b2}	p_{a2}	Π_{MM} ×1000	Vol(I)	Vol(L)	W ×1000	dU ×1000
W_{min}	.625	.625	.792, .809	.870, .908	.908	7.402	.183	.312	5.687	−.040
I	.631	.631	.792	.854	.925	0	0	0	22.452	−.123
II	.631	.631	.792	.864	.914	15.658	0	.316	8.574	−.091
III	.631	.631	.792	.874	.903	18.022	.316	.316	6.582	0
IV	.621	.621	.805	.904	.904	0	.311	.311	6.082	−.004
V	.626	.626	.792, .805	.875, .903	.903	8.996	.313	.313	6.328	−.002
VI	.622	.622	.792, .815	.864, .914	.914	0	0	.311	8.206	−.093
U_{max}	.631	.631	.792	.874	.903	18.022	.631	.631	6.582	0
A	.631	.631	.792	.858	.921	7.402	0	.118	15.880	−.118
B	.625	.625	.800	.891	.903	7.402	.625	.625	6.285	−.002

The parameter values are $X = 1$, $\beta = .90$, $q = .5$, $Y_1 = 100$, $Y_2^S = 98$, $Y_2^B = 102$, $Y_3^S = 102$, $Y_3^B = 98$, $k = 1$. Points I, II,..., VI are the endpoints in Figure 1. W_{min} is the optimum point (denoted by ∇ in Figure 1), which minimizes the objective function (9). $dU = U - U_{max}$ is the agents' expected utility relative to its maximum point, where U_{max} is the optimal point from the agents' perspective. At points A and B the market maker's revenue equals the market maker's revenue at the optimum (W_{min}), namely 7.402.

$\leq p^L_{a1}$), and hence the period-2 bid prices of the two bonds possess the same relation ($p^I_{b2} \leq p^L_{b2}$).

Information on the six endpoints of the equilibria marked in Figure 1, as well as other equilibria that are analyzed later, are contained in Table 3. Π_{MM} is gross per capita market-maker revenue, Vol(L) and Vol(I) are the per capita volume of trade in period 2 of the liquid and the illiquid bonds, $k\Omega$ and Ω are the per capita supply of liquid and illiquid bonds, and W is the value of the objective function.

Considering all the possible equilibria, the one that minimizes the objective function appears at the top of Table 3. In this equilibrium both bonds are traded, and, although the face value of bonds issued is higher than in some other cases, this is more than offset by the level of market-maker revenues. The intuition behind this result is that the issuer engages in discriminatory pricing in order to extract consumer surplus. A bond consists of two components: the claim to the final payoff and the option to resell prior to maturity (i.e., in period 2). The issuer segments markets across these two components.

At the optimum the liquid bonds are perfectly liquid ($p_{a2} = p^L_{b2}$). Foreseeing this, in period 1, investors have the option to purchase a liquid bond, which can be traded in period 2 with no transaction costs, or an illiquid bond, which carries transaction costs in period 2. The liquidity services provided by the liquid bond make it more valuable in period 1. At the optimum point (and all other points in Equilibrium 5), sellers sell all of their holdings of the liquid bond (for which there is a higher bid price posted) and a fraction of their illiquid bonds.

The prices of the two bonds at the optimum can be used to quantify the value of liquidity. The price of the illiquid bond is exactly the value of a bond that provides a claim to period-3 consumption and that cannot be traded in period 2 (i.e., $p^L_{a1} = \beta^2 E_1[u'(c_3)/u'(c_1)] = .792$). Agents do not pay for the liquidity provided by this bond, even though it trades in period 2, because the marginal illiquid bond is not traded.[11] In contrast, the price of the liquid bond reflects both its value as a claim on period-3 consumption and its value as a means of portfolio adjustment in period 2. The value of liquidity is simply the difference between the period-1 ask prices of the two bonds (i.e., $p^L_{a1} - p^I_{a1} = .016$, which is 2 percent of the value of the bond). In Section 3.3 we show that when the issuer can also choose the relative supplies of the bonds, more consumer surplus can be extracted. By increasing the supply of the liquid bond, unpriced liquidity services that were provided by the illiquid bond are now priced.

Is the absence of a one-period security crucial in any way? Recall that at the optimum the liquid bond is perfectly liquid in period 2. Consequently, in period 1 it trades at the same price as a claim for period-2 consumption (i.e., $p^L_{a1}/p^L_{b2} = \beta E_1[u'(c_2)/u'(c_1)] = .891$). Therefore, the two-period bond issued in period 1 provides the same set of possible consumption streams as two one-period bonds issued in period 1 and refinanced in period 2.[12] Nevertheless, the return spread between the liquid and illiquid bonds is not a term premium in the traditional sense of compensating for interest rate risk, because interest rates are nonstochastic in our model.

To illustrate the trade-offs between the liquidity services provided by the bonds, the face value of the bonds, and the expected utility of the agents in the segmented and unsegmented equilibria, consider points A and B in Table 3 and the optimum point $W_{\min}$. To highlight these trade-offs, market-maker revenue is held constant across these three points. Since total revenue in period 1 is held constant, variations in the ask prices of the bonds are reflected in changes in the face value. Points A and B are both unsegmented equilibria, differing in the amount of period-2 trade. At point A, the sellers sell only a portion of their holdings. The period-1 ask price, therefore, reflects only the value of a claim on period-3 consumption (i.e., $p_{a1} = \beta^2 E_1[u'(c_3)/u'(c_1)] = .792$). Since liquidity is not valued, the ask price is relatively low, hence the relatively high face value. Expected utility is low because of a lack of consumption smoothing. At point B, the

[11] The period-1 ask price of the illiquid bond (p^I_{a1}) is approximately equal for all the points except IV. More generally, at every feasible point below the line defined by points III and V this price is exactly the value of a two-period security that will be held to maturity. Whenever buyers and sellers hold some of the illiquid bonds to maturity, this result holds.

[12] More generally, any combination of perfectly liquid two-period bonds and one-period bonds, holding total supply constant, will generate the same results.

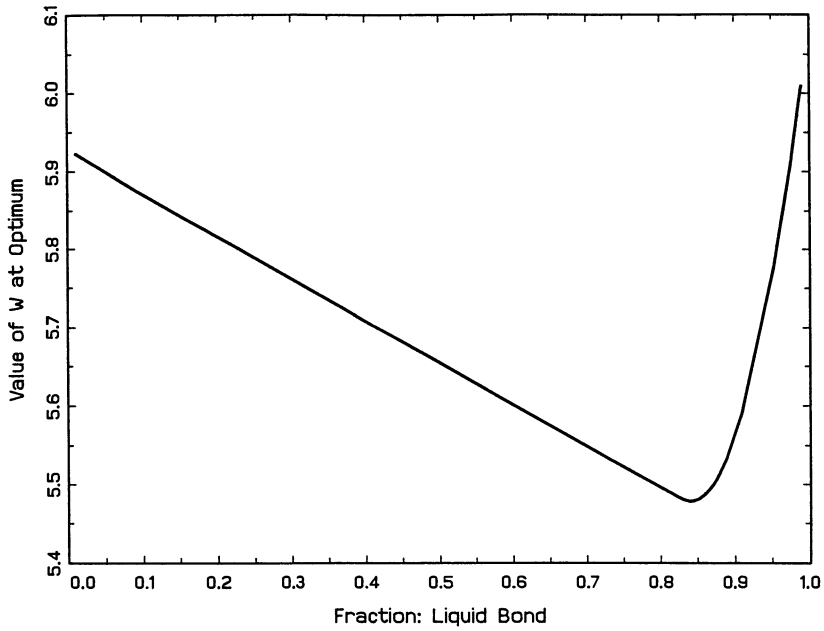

Figure 2
Minimum value of the objective function (×1000) versus relative supply of the liquid bond
The minimized value of the objective function versus the fraction of liquid bonds in the total supply [$k/(1 + k)$]. Other parameter values are as in Figure 1.

sellers sell all their bonds in period 2. The period-1 ask price, therefore, also reflects the value of liquidity (i.e., $p_{a1} = .800 > \beta^2 E_1[u'(c_3)/u'(c_1)] = .792$). Supplying liquidity raises the ask price and decreases the face value while aiding consumption smoothing and increasing expected utility. At the optimum point $W_{\min}$, the issuer is able to further increase the period-1 ask price of the liquid bond with a smaller offsetting decrease in the ask price of the illiquid bond. As a result, expected utility decreases.[13]

3.3. The effect of changing the relative supplies

The foregoing analyses are conducted under the assumption that the relative supply of the two bonds is equal. We now extend the issuer's choice space to include the relative supply of each type of bond (given the fixed total revenue). In Figure 2 the minimized value of the objective function is plotted against the supply of the liquid bond as a fraction of the total supply, $k/(1 + k)$. The global optimum is achieved for $k^* \approx 5$. It is not surprising that the global optimum

[13] The decrease in expected utility is also attributable to a reduction in period-2 trade and the resultant decrease in consumption smoothing, but holding trade constant and moving from unsegmented to segmented markets also decreases expected utility, as a comparison of points III and IV illustrates.

occurs at a point where $k > 1$ given the results in Table 3. As noted, the price of the illiquid bond reflects its value as a two-period security, not as a source of liquidity. This bond does provide liquidity services, however, since it is traded in period 2. In contrast, the global optimum occurs at a point where all liquidity needs are supplied by the liquid bond; hence, agents pay for all the liquidity they use. As the demand for, and the value of, liquidity decreases, the value of k at the global optimum decreases. Therefore, since endowment volatility and the value of liquidity are positively related (as demonstrated in the next section), the issuer would choose to issue proportionately fewer liquid securities in less volatile environments.

Market-maker revenue is zero at the global optimum because the liquid bond is perfectly liquid and the illiquid bond does not trade. Consequently, the ability of the issuer to extract market-maker revenue is no longer required in order to motivate the objective function. Moreover, this equilibrium is the same as one in which a sequence of two one-period bonds replaces the liquid bond, so market making is also no longer necessary at the global optimum.

The fact that only the liquid bond is traded in the second period at the global optimum is an artifact of the structure of the uncertainty in the model. Although none of the agents know in period 1 what their period-2 and period-3 endowments will be, there is no aggregate uncertainty in the economy. In other words, the aggregate endowment and the aggregate need for liquidity in period 2 are known for certain at time 1. The volume of period-2 trade is also known, and the only question is which group of agents will be on the buy side and which on the sell side.

3.4. Volatility and the value of liquidity

While the endowment stream is not a choice variable in the issuer's or the agents' optimization problem, it is interesting to consider the effect of endowment volatility on the value of liquidity. By necessity we conduct an exercise in comparative statics. Endowment uncertainty in our model is of particular interest because there is no aggregate uncertainty. Nevertheless, there is a type uncertainty in period 1, which is a function of the endowment stream. Does the uncertainty at the "micro" (agent) level affect the value of liquidity? In cases where the representative agent paradigm holds, the answer is no. In our model, market incompleteness, in the form of a short-sale constraint joint with heterogeneous agents and the assumed form of market making, results in an economy where the value of liquidity increases with microlevel uncertainty.

In order to study this link between the volatility of endowments and the value of liquidity, we define endowment variability to be

$$V = q[(Y_2^S - Y_2)^2 + (Y_3^S - Y_3)^2] + (1 - q)[(Y_2^B - Y_2)^2 + (Y_3^B - Y_3)^2], \quad (10)$$

where Y_2 and Y_3 are the fraction-weighted averages as defined in Section 3. The ask price differential in period 1, $p_{a1}^L - p_{a1}^I$, serves as the measure of the relative value of liquidity.

In Table 4 we conduct a comparison across economies, letting the variability of agents' endowment processes change while keeping the mean of the endowments fixed. The value of liquidity increases in the variability of endowments, from .006 (.8 percent of the price of the liquid bond) for $V = 2.0$ to .016 (2 percent) for $V = 8.0$.

Notice that the objective function minimized by the issuer (denoted by W) is decreasing in the variability of endowments. This relation is due to the value agents attach to income smoothing. The higher the variability of endowments, the more the issuer can exploit the need for liquidity.

Empirically, there is an interesting observed link between liquidity and interest rate volatility, as suggested by Sargen et al. (1986) for the JGB market and as demonstrated by Kamara (1990) and Amihud and Mendelson (1991) for the U.S. Treasury market. An increased liquidity premium, quantified by the yield spread between liquid and illiquid securities, is common in periods of high interest rate volatility. While there is no precise analog to interest rate volatility in our model (interest rates are nonstochastic), one could think of interest rate volatility as resulting from both aggregate and microlevel variability in endowments. In a multiperiod exchange economy with perfect markets, stochastic volatility at the aggregate level is positively related to interest rate volatility [Boudoukh (1992)]. The link between micro level variability and interest rate volatility requires an extension of our model, which exploits market incompleteness to generate static price and interest rate effects, to a dynamic setting.

4. The Market Maker as a Maximizer

In Section 1.3 we preclude competitive market making as a framework in which segmentation may arise. In Section 3 we consider a regulated monopolist, and liquidity is priced via price segmentation. We now consider the case of a market maker who acts independently in the second period so as to maximize his own revenue. We demonstrate that segmentation may not be optimal when the market maker is an unregulated monopolist.

The market maker solves an optimization problem that involves determining the two bid prices in the second period (p_{b2}^L and p_{b2}^I).

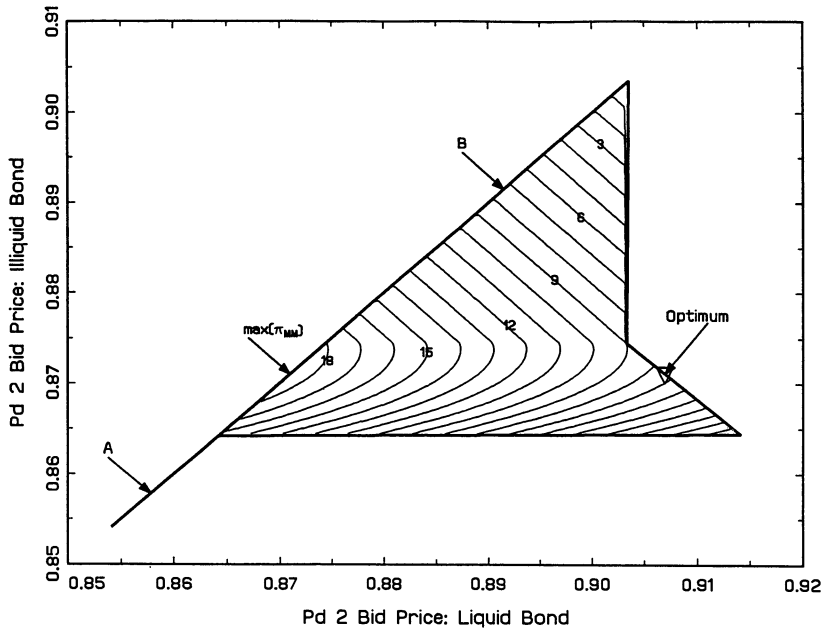

Figure 3
Fixed market-maker revenue contours
Fixed market-maker revenue contours ($\Pi_{MM} \times 1000$). Parameter values are as in Figure 1. The optimum point minimizes the objective function $W \times 1000$. max(Π_{MM}) is the point where the market maker's revenue is maximized. Points *A* and *B* have the same Π_{MM} as at the optimum. Details regarding volume, market-maker revenue, the value of the objective function, and the agents' utility are in Table 3.

The period-2 ask price of both bonds (p_{a2}) is then determined in equilibrium. The problem can be formulated as follows:

$$\max_{p_{b2}^L, p_{b2}^I} (p_{a2} - p_{b2}^L)\text{Vol}(L) + (p_{a2} - p_{b2}^I)\text{Vol}(I).$$

The problem is prohibitively complicated algebraically, and we resort to a numerical analysis using the base case parameters presented earlier.

In Figure 3, we present fixed market-maker revenue contours within the feasible price range. The optimal point for the market maker is at the "top-of-the-hill" on the 45° line, where there is no segmentation. This is in contrast to the issuer's optimal point, which we earlier showed exhibits segmentation. Market-maker revenue is maximized at a value of .0187 when the period-1 ask price is .792, and the period-2 bid and ask prices are .871 and .907, respectively.

In order to gain intuition for the generality of this result, that segmentation is not optimal, note that the volume of trade in both

Table 4
Uncertainty and the value of liquidity

Y_2^S	Y_3^S	Y_2^B	Y_3^B	V	$p_{a1}^L - p_{a1}^I$	W ×1000
98.0	102.0	102.0	98.0	8.0	.016	5.657
98.5	101.5	101.5	98.5	4.5	.012	13.135
99.0	101.0	101.0	99.0	2.0	.006	18.682

The parameter values are $X = 1$, $\beta = .90$, $q = .5$, $Y_1 = 100$, $k = 1$. V is a measure of endowment volatility, defined in (10).

bonds and the period-2 ask price are all functions of the two period-2 bid prices over which the market maker optimizes. Consequently, the effect on market-maker revenue of a change in the bid prices may be ambiguous. However, when endowments are much larger than the face value of the bonds, as in our numerical example, it is possible to trace the lack of segmentation at the optimum to the relative magnitude of the partial derivatives in the first-order conditions of the market maker's problem.

Observe that in Figure 3 the market maker's revenue increases as the liquid bond's bid price decreases (which amounts to moving horizontally to the left). The dominant effect is an increase in the spread and, hence, an increase in revenue. The induced changes in the period-2 ask price and the volume of trade are small.[14] The optimum is, therefore, a symmetric equilibrium. The symmetric equilibria are defined by the period-2 spread on the bonds. As the spread increases, the volume of trade decreases, and the optimum is the best trade-off between these offsetting effects.

5. Conclusions

We show that in an economy with assets with identical payoffs it is possible to sustain an equilibrium in which one asset is liquid and the other illiquid. In other words, divergent bid–ask spreads and prices for similar assets can be supported in equilibrium. Furthermore, if such segmentation is a decision variable, then it may be optimally chosen over a homogeneously liquid market structure. The

[14] More formally,

$$\frac{\partial p_{a2}}{\partial p_{b2}^L} \ll 1, \quad \frac{\partial \mathrm{Vol}(L)}{\partial p_{b2}^L}(p_{a2} - p_{b2}^L) \ll \mathrm{Vol}(L),$$

$$\frac{\partial \mathrm{Vol}(I)}{\partial p_{b2}^L}(p_{a2} - p_{b2}^L) \ll \mathrm{Vol}(L).$$

degree of segmentation and the relative supplies of liquid and illiquid assets, if these are choice variables, will be determined by the parameters of the economy such as the volatility of the endowment stream.

It is important to note that our results are sustainable only under specific assumptions about short sales, market making, and so on, that match fairly closely the institutional features of the JGB market. To the extent that such conditions are absent, the effects documented should be smaller or take different forms. The liquidity effect in the U.S. Treasury market, for example, is constrained by the lower cost of short selling.

Our results are derived without asymmetric information, market-maker inventory, search costs, or ex ante heterogeneous agents. Similar results can be achieved with ex ante heterogeneous agents. The separation is between "speculators" (those who need to trade frequently) and "investors" (those who have little need to trade prior to maturity). In the JGB market we can identify investors (e.g., insurance companies and trust banks who buy and hold bonds to match long-term liabilities) and speculators (e.g., securities firms and city banks who sometimes engage in hectic speculation in government bonds), a distinction that seems to be absent from this model. Here agents play the role of both investors and speculators as they purchase a portfolio of liquid and illiquid assets. It is plausible, nevertheless, that the dichotomy between speculators and investors, and the identification of these with certain institutions, is an artifact of their function and specialization in a system in which they are simply the intermediaries who represent agents in the economy. From this perspective, the institutions' activities are just an execution of the trading behavior implied in our model.

Several questions regarding the evolution of the liquidity premium over time are impossible to address in our three-period model. One interesting phenomenon in the JGB market is that the benchmark spread on a particular benchmark issue does not decline smoothly as the designation of a new benchmark bond approaches. Instead, the spread is relatively stable until it drops sharply a few weeks before the changeover. A second challenge is to explain the variability of the liquidity premium and link it to fundamentals. Finally, what is the prospect for the future of the benchmark effect? A decline in its magnitude is anticipated as the Japanese short-term market develops, short-sale constraints disappear, market-making changes form, and domestic government bond futures markets gain liquidity. Preliminary evidence can be found in Boudoukh and Whitelaw (1991).

Appendix A: Proof of Theorem 1

First we show the following lemma.

Lemma 1. *If the agents can costlessly sell bonds short in period 2 at the bid price (without any restrictions), then in equilibrium (i) if trade occurs in both bonds in period 2, then* $p^I_{b2} = p^L_{b2}$ *and* $p^I_{a1} = p^L_{a1}$*; (ii) if no trade occurs in period 2, then* $p^I_{a1} = p^L_{a1}$*; and (iii) if in period 2 the illiquid bond is not traded and the liquid bond is traded, then* $p^I_{b2} \leq p^L_{b2}$ *and* $p^I_{a1} = p^L_{a1}$.

Proof. (i) First, suppose that bid prices are not equal in period 2. For each bond sold the agent receives additional units of consumption equal to the bond's bid price and gives up one unit of consumption in period 3. This trade-off is identical regardless of whether the sale requires the agent to go short, because short positions can be closed out in the final period at a cost of one unit of consumption (the bid and ask prices of both bonds at maturity). Consequently, agents strictly prefer to sell the bond with the higher bid price. Since markets must clear in period 2, this contradicts the assumption of positive trade in both bonds. The equality of bid prices ensures that each bond has identical value to a purchaser in period 1. Hence, in a rational expectations equilibrium, period-1 ask prices must also be identical.

(ii) Given that no trade occurs in period 2, each bond is simply a claim on one unit of consumption in period 3. As a result, the bonds must have identical ask prices in period 1.

(iii) First, suppose that $p^I_{b2} > p^L_{b2}$. We saw that this implies all the trade in period 2 will be in the illiquid bond, which contradicts our assumption. If $p^I_{b2} = p^L_{b2}$, then the logic of (i) still holds and $p^I_{a1} = p^L_{a1}$. If $p^I_{b2} < p^L_{b2}$, it appears that $p^I_{a1} < p^L_{a1}$ would be possible, the logic being that since the liquid bond can be sold for more in period 2, the agents will be willing to pay more for it in period 1. The fallacy of this argument can be seen by considering the following strategies: (a) buy one liquid bond in period 1 and sell it in period 2, or (b) buy one illiquid bond in period 1, sell one liquid bond short in period 2, and close out the short position in period 3 using the payoff from the illiquid bond. The second strategy clearly dominates the first. In other words, since agents are indifferent between going short and liquidating holdings at any given bid price, they are not willing to pay for liquidity that they can achieve through short sales.

Theorem 1 then follows from the elimination rule for equivalent equilibria. ∎

Appendix B: Solving Equilibrium 5

Under the assumption that this class of equilibria occurs and that the agents are carrying holdings h^I_1 and h^L_1 into period 2, the period 2 problem of the sellers (P2S), assuming log utility, has the solution

$$h_{S2}^{I*} = \frac{\beta}{1+\beta}\frac{Y_2^S + h_1^L p_{b2}^L}{p_{b2}^I} + \frac{1}{1+\beta}(\beta h_1^I - Y_3^S). \tag{B1}$$

Under the same assumptions, the solution to the period 2 problem of the buyers (P2B) is

$$h_{B2}^{*} = \frac{\beta}{1+\beta}\left[\frac{Y_2^B}{p_{a2}} + h_1^L + h_1^I\right] - \frac{Y_3^B}{1+\beta}. \tag{B2}$$

Substituting these solutions back into the initial period problem we obtain (P1). The solutions for the optimal values of h_1^I and h_1^L are defined implicitly by the following two equations:

$$\begin{aligned}\frac{p_{a1}^I}{Y_1 - h_1^I p_{a1}^I - h_1^L p_{a1}^L} = {} & \frac{\beta q p_{a2}(1 - \beta/(1+\beta))}{Y_2^B - (h_{B2}^{*} - (h_1^I + h_1^L))p_{a2}} \\ & + \frac{\beta(1-q)p_{b2}^I(1-\beta/(1+\beta))}{Y_2^S + h_1^L p_{b2}^L - (h_{S2}^{I*} - h_1^I)p_{b2}^I} \\ & + \frac{\beta^2 q\beta/(1+\beta)}{Y_3^B + b_{B2}^{*}} + \frac{\beta^2(1-q)\beta/(1+\beta)}{Y_3^S + h_{S2}^{I*}},\end{aligned} \tag{B3}$$

$$\begin{aligned}\frac{p_{a1}^L}{Y_1 - h_1^I p_{a1}^I - h_1^L p_{a1}^L} = {} & \frac{\beta q p_{a2}(1 - \beta/(1+\beta))}{Y_2^B - (h_{B2}^{*} - (h_1^I + h_1^L))p_{a2}} \\ & + \frac{\beta(1-q)(p_{b2}^L - [\beta/(1+\beta)](p_{b2}^L/p_{b2}^I)p_{b2}^I)}{Y_2^S + h_1^L p_{b2}^L - (h_{S2}^I - h_1^I)p_{b2}^I} \\ & + \frac{\beta^2(1-q)\beta/(1+\beta)}{Y_3^B + b_{B2}^{*}} \\ & + \frac{\beta^2(1-q)[\beta/(1+\beta)]p_{b2}^L/p_{b2}^I}{Y_3^S + h_{S2}^{I*}}.\end{aligned} \tag{B4}$$

In equilibrium, for supply to equal demand in each period, (1)–(3) must also hold. The seven Equations (1)–(3) and (B1)–(B4), define the equilibrium, which has nine variables: h_1^I, h_1^L, h_{B2}, h_{S2}^I, p_{a1}^I, p_{a1}^L, p_{a2}, p_{b2}^I and p_{b2}^L. Using appropriate substitutions, we can reduce the problem to three equations in five unknowns. Substituting (1) and (2) into (B1) and (B2) and the resulting equations into (3) and rearranging yields

$$\frac{q\beta Y_2^B}{p_{a2}} = (1+k)\Omega + \beta(1-q)k\Omega + (1-q)Y_3^S$$

$$+ qY_3^B - \frac{\beta(1 - q)(Y_2^S + k\Omega p_{b2}^L)}{p_{b2}^I}. \tag{B5}$$

Substituting (1) and (2) into (B3), simplifying, and combining terms, we get

$$\frac{p_{a1}^I}{Y_1 - \Omega(p_{a1}^I + kp_{a1}^L)} = \frac{\beta(1 + \beta)qp_{a2}}{Y_2^B + [Y_3^B + (1 + k)\Omega]p_{a2}} + \frac{\beta(1 + \beta)(1 - q)p_{b2}^I}{Y_2^S + k\Omega p_{b2}^L + (Y_3^S + \Omega)p_{b2}^I}. \tag{B6}$$

Similar algebra using (B4) produces

$$\frac{p_{a1}^L}{Y_1 - \Omega(p_{a1}^I + kp_{a1}^L)} = \frac{\beta(1 + \beta)qp_{a2}}{Y_2^B + [Y_3^B + (1 + k)\Omega]p_{a2}} + \frac{\beta(1 + \beta)(1 - q)p_{b2}^L}{Y_2^S + k\Omega p_{b2}^L + (Y_3^S + \Omega)p_{b2}^I}. \tag{B7}$$

The system of equations (B5)–(B7) simplifies to (4)–(6).

References

Admati, A. R., and P. Pfleiderer, 1988, "A Theory of Intraday Patterns: Volume and Price Variability," *Review of Financial Studies,* 1, 3–40.

Amihud, Y., and H. Mendelson, 1980, "Dealership Market: Market-Making with Inventory," *Journal of Financial Economics,* 8, 31–53.

Amihud, Y., and H. Mendelson, 1986, "Asset Pricing and the Bid–Ask Spread," *Journal of Financial Economics,* 17, 223–249.

Amihud, Y., and H. Mendelson, 1991, "Liquidity, Maturity, and the Yields on U.S. Treasury Securities," *Journal of Finance,* 46, 1411–1425.

Boudoukh, J., 1992, "An Equilibrium Model of Nominal Bond Prices with Inflation–Output Correlation and Stochastic Volatility"; forthcoming in *Journal of Money, Credit and Banking.*

Boudoukh, J., and R. Whitelaw, 1991, "The Benchmark Effect in the Japanese Government Bond Market," *Journal of Fixed Income,* 1(2), 52–59.

Foster, F. D., and S. Viswanathan, 1990, "A Theory of the Interday Variations in Volume, Variance, and Trading Costs in Securities Markets," *Review of Financial Studies,* 3, 593–624.

Garbade, K., and W. Silber, 1976, "Price Dispersion in the Government Securities Market," *Journal of Political Economy,* 84, 721–740.

Glosten, L., and P. Milgrom, 1985, "Bid, Ask and Transaction Prices in a Specialist Market with Heterogeneously Informed Traders," *Journal of Financial Economics,* 14, 71–100.

Grossman, S., and G. Laroque, 1990, "Asset Pricing and Optimal Portfolio Choice in the Presence of Illiquid Durable Consumption Goods," *Econometrica,* 58, 25–51.

Ho, T., and H. Stoll, 1981, "Optimal Dealer Pricing Under Transaction and Return Uncertainty," *Journal of Financial Economics,* 9, 47–73.

Ho, T., and H. Stoll, 1983, "The Dynamics of Dealer Markets Under Competition," *Journal of Finance,* 38, 1053–1074.

Kamara, A., 1990, "Liquidity and Short-Term Treasury Yields," working paper, University of Washington.

Lippman, S., and J. McCall, 1986, "An Operational Measure of Liquidity," *American Economic Review,* 76, 43–55.

Mason, R., 1987, "The Yen Bond Markets," Credit Suisse First Boston, CSFB Research.

Pagano, M., 1989, "Trading Volume and Asset Liquidity," *Quarterly Journal of Economics,* 103, 255–274.

Sargen, N., K. Schoenholtz, S. Blitz, and S. Elhabashi, 1986, "Trading Patterns in the Japanese Government Bond Market," Salomon Brothers Inc., Bond Market Research.

Tuckman, B., and J. L. Vila, 1992, "Arbitrage with Holding Costs: A Utility-Based Approach," *Journal of Finance,* 47, 1283–1302.